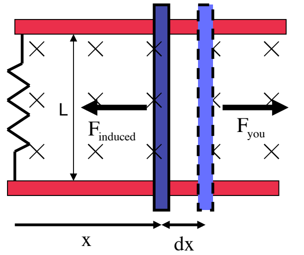
Problem 31.23

What force is required to keep the bar moving with a velocity of 2 m/s?



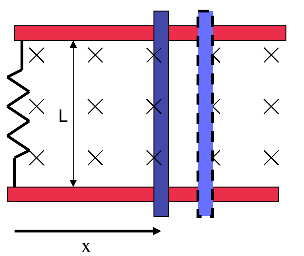
$R = 6\ \Omega$, $L = 1.2$ m and

$B = 2.5$ Teslas into the page.

The bar is one side of a rectangular "coil" the other sides of which are the resistor section and the two rails. As the bar move to the right, the magnetic flux through the "coil" increases. The increasing flux motivates and induced current which interacts with the external field to produce a force that fights what's happening. The acceleration is zero, so the induced and applied forces will add to zero. To get the induced force, you need to determine the induced EMF through the coil, the current generated due to that EMF, then use iLxB to determine the force.

ALWAYS begin a Faraday's Law problem by generating a GENERAL EXPRESSION for the flux through the coil. For this case:

$$\Phi_B = \vec{B} \bullet \vec{A}$$
$$= B\ A \cos 0^\circ$$
$$= B\ (Lx)$$



$R = 6\ \Omega$, $L = 1.2$ m and

$B = 2.5$ Teslas into the page.

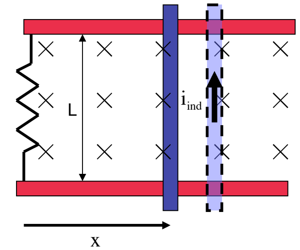
From Faraday's Law, the induced EMF is:

$$\varepsilon_{induced} = -N \frac{d\phi_B}{dt}$$
$$= -(1) \frac{d(BLx)}{dt}$$
$$= -\ BL \frac{dx}{dt}$$
$$= -\ BL\ v$$

(we'll use the magnitude from here on)

The induced current generated by the EMF is:

$$\varepsilon_{induced} = i\ R$$
$$\Rightarrow\ BLv = i\ R$$
$$\Rightarrow\ i = \frac{BLv}{R}$$



$R = 6\ \Omega$, $L = 1.2$ m and

$B = 2.5$ Teslas into the page.

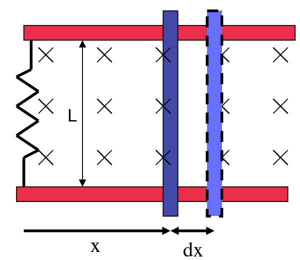
The induced current's induced force is:

$$F_{induced} = i\ \vec{L} x \vec{B}$$
$$= \left(\frac{BLv}{R}\right) LB \sin 90^\circ$$
$$= \frac{L^2 B^2 v}{R}$$
$$= \frac{(1.2\ m)^2 (2.5\ T)^2 (2\ m/s)}{(6\ \Omega)}$$
$$= 3\ N$$

As the induced field's force is FIGHTING the move to the right (i.e., it is to the left), the force you apply must be to the RIGHT!

b.) The rate at which energy is provided to anything is POWER related.

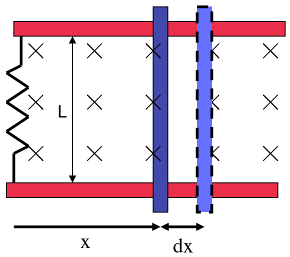
The power provided to the circuit will be the same as the power dissipated by the resistor. That is:



$$P_{induced} = i^2\ R$$
$$= \left(\frac{BLv}{R}\right)^2 R$$
$$= \frac{B^2 L^2 v^2}{R}$$
$$= \frac{(2.5\ T)^2 (1.2\ m)^2 (2\ m/s)^2}{(6\ \Omega)}$$
$$= 6\ W$$

EXTRA (for those who care)

If you hadn't made the connection between power and the energy dissipated by the resistor, you could have used the definition of work and power and seen where it took you. Doing so yields:



$$P = \frac{dW}{dt} = \frac{\vec{F} \bullet d\vec{x}}{dt}$$

$$= \vec{F} \bullet \frac{d\vec{x}}{dt}$$

$$= Fv$$

$$= \left(\frac{B^2 L^2 v}{R} \right) v$$

No surprise--same conclusion!